SEAC
Space Environment to Anomaly Correlator

Dr. Jane M. Burward-Hoy
Principal Investigator
Mr. Tyler Krzykowski
Lead Developer
Dr. James Kestyn
Developer
Reece Broughton
Developer
Dr. Irene Budianto-Ho
Aerospace Engineer

Dr. Gordon R. Wilson
Program Manager, AFRL/RV

Approved for public release; distribution is unlimited. Public Affairs release approval #: AFMC-2020-0191.

Motivation

- Space assets experience anomalous behaviors ("anomaly events")
- Operators would like to understand:
  - Whether anomaly events are due to space environment hazards
  - The type of environment a space asset of interest is susceptible to

There is a need for a tool that identifies the strongest correlation between anomaly events and the space environment

The Anomaly Attribution Process

Identify the right environmental "driver"
- Causative agent of the reported anomalous events (if any)

↓

Identify the strongest correlation
- Tailor energy range and/or time interval of driver

↓

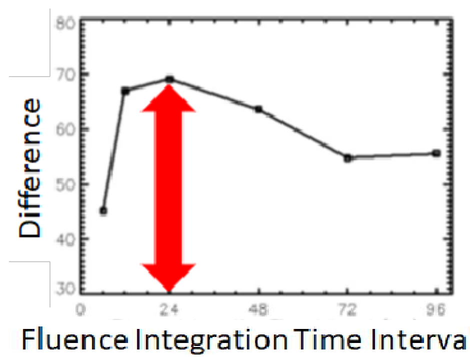
Calculate the ratio of the instantaneous to mission-averaged likelihood of an anomaly due to the driver
- Defines a Hazard Quotient Rule

U.S. AIR FORCE

stellar science

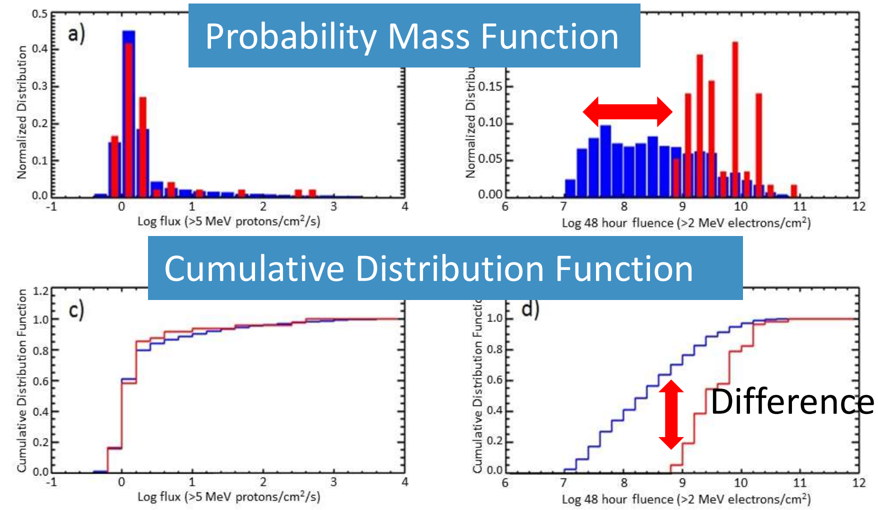
The Process Applied Manually

- Exploit statistical methods (O'Brien [Ref. 1])
- Apply method to a communication satellite ("target")
  - Sensor mode switching anomaly in Wrenn [Ref. 2] and Wrenn [Ref. 3]
- Use GOES-7 EPS environmental data
  - Separated from target by ~90 deg longitude
- Construct Anomaly flux distribution
  - Match anomaly times to candidate driver distribution times
- Compare Electron and Proton candidate drivers:
  - Compute Cumulative Difference Functions
  - Quantify differences
  - Vary fluence integration time interval



Protons > 5 MeV (instantaneous flux)

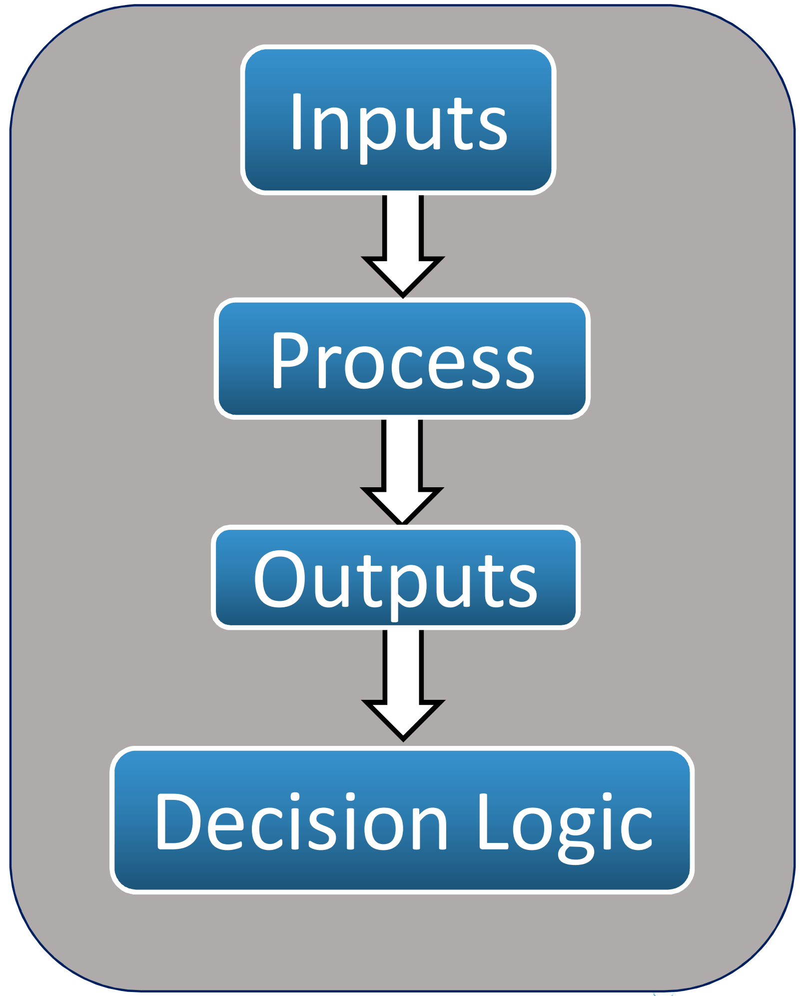
Electrons > 2 MeV (48-hour fluence)



"Likely" Driver Electrons > 2 MeV
24-hour fluence integration time interval
(indicates internal charging hazard)

SEAC Overview

- SEAC automates this manual process
- Configurable parameters in a single input JSON file
  - Fluence intervals, particle types
  - Input Environment Data
    - GOES, SCATHA, CRRES
    - GeoSu Model for electrons > 2 MeV (Ref. [4])
  - Statistical Methods
    - Kolmogorov-Smirnov, Anderson-Darling, Mean Difference, etc.
  - Output options (Hazard Quotients, diagnostics data)
- Internal Decision logic
  - Permutation testing (p-value) and FWER and FDR (UC Berkeley Stats Dept)
- Standalone execution
  - Command Line Interface (CLI)



SEAC in Action (1/4): The Inputs

- Path to environmental data
- Analysis methods
- Path for outputs
- Electron mapping option (GeoSu Model)
- Environmental candidate drivers and desired fluence intervals
- Either mission lifetime duration or full climatology
- A thread count
- Anomaly listing
- Data target locations over mission duration


```
"ecpDataPath" : "satdat.ngdc.noaa.gov",
"analysisTypes" : ["KOLMOGOROV_SMIRNOV_TEST"],
"driverOutputPath" : "wrenn/test",
"writeDriverOutputData" : true,
"mapElectronChannels" : false,
"driverAnalysisResults" : true,
"bootstrapIterations": 500,
"computeHazardQuotient": true,
"enabledDriverTypes": [ "ELECTRON", "PROTON" ],
"electronFluenceIntervalInHours": [18,24,36,48],
"protonFluenceIntervalInHours": [36,48,72,96],
"missionLifetime": { "start": "1991-01-01 00:00:00.000", "end": "1994-12-31 23:59:00.000" },
"threadCount": 8,
"anomalies" : [
  { "type" : "1", "targetId" : "GOES-07", "startDateTime" : "1991-03-28 13:49:00.000", "endDateTime" :"1991-03-28 13:49:00.000"},
  { "type" : "1", "targetId" : "GOES-07", "startDateTime" : "1991-03-30 09:55:00.000", "endDateTime" :"1991-03-30 09:55:00.000"},
  { "type" : "1", "targetId" : "GOES-07", "startDateTime" : "1992-05-12 13:29:00.000", "endDateTime" :"1992-05-12 13:29:00.000"},
  { "type" : "1", "targetId" : "GOES-07", "startDateTime" : "1992-05-15 08:43:00.000", "endDateTime" :"1992-05-15 08:43:00.000"},
  { "type" : "1", "targetId" : "GOES-07", "startDateTime" : "1992-07-25 13:27:00.000", "endDateTime" :"1992-07-25 13:27:00.000"},
  { "type" : "1", "targetId" : "GOES-07", "startDateTime" : "1992-08-13 11:51:00.000", "endDateTime" :"1992-08-13 11:51:00.000"},
  { "type" : "1", "targetId" : "GOES-07", "startDateTime" : "1992-09-05 11:51:00.000", "endDateTime" :"1992-09-05 11:51:00.000"},
  { "type" : "1", "targetId" : "GOES-07", "startDateTime" : "1992-09-07 12:03:00.000", "endDateTime" :"1992-09-07 12:03:00.000"},
  { "type" : "1", "targetId" : "GOES-07", "startDateTime" : "1992-09-20 10:43:00.000", "endDateTime" :"1992-09-20 10:43:00.000"},
  { "type" : "1", "targetId" : "GOES-07", "startDateTime" : "1992-10-02 03:53:00.000", "endDateTime" :"1992-10-02 03:53:00.000"},
  { "type" : "1", "targetId" : "GOES-07", "startDateTime" : "1992-10-03 10:25:00.000", "endDateTime" :"1992-10-03 10:25:00.000"},
  { "type" : "1", "targetId" : "GOES-07", "startDateTime" : "1992-10-17 10:28:00.000", "endDateTime" :"1992-10-17 10:28:00.000"},
  { "type" : "1", "targetId" : "GOES-07", "startDateTime" : "1992-11-15 12:19:00.000", "endDateTime" :"1992-11-15 12:19:00.000"},
  { "type" : "1", "targetId" : "GOES-07", "startDateTime" : "1992-12-13 11:50:00.000", "endDateTime" :"1992-12-13 11:50:00.000"},
  { "type" : "1", "targetId" : "GOES-07", "startDateTime" : "1993-01-05 10:30:00.000", "endDateTime" :"1993-01-05 10:30:00.000"},
  { "type" : "1", "targetId" : "GOES-07", "startDateTime" : "1993-01-08 13:26:00.000", "endDateTime" :"1993-01-08 13:26:00.000"},
  { "type" : "1", "targetId" : "GOES-07", "startDateTime" : "1993-02-03 09:18:00.000", "endDateTime" :"1993-02-03 09:18:00.000"},
  " : "1", "targetId" : "GOES-07", "startDateTime" : "1993-02-11 09:46:00.000", "endDateTime" :"1993-02-11 09:46:00.000"},
  : "1", "targetId" : "GOES-07", "startDateTime" : "1993-02-24 13:24:00.000", "endDateTime" :"1993-02-24 13:24:00.000"},
  : "1", "targetId" : "GOES-07", "startDateTime" : "1993-03-18 11:05:00.000", "endDateTime" :"1993-03-18 11:05:00.000"},
  : "1", "targetId" : "GOES-07", "startDateTime" : "1993-03-26 10:49:00.000", "endDateTime" :"1993-03-26 10:49:00.000"},
  : "1", "targetId" : "GOES-07", "startDateTime" : "1993-04-11 10:54:00.000", "endDateTime" :"1993-04-11 10:54:00.000"},
  : "1", "targetId" : "GOES-07", "startDateTime" : "1993-04-22 11:10:00.000", "endDateTime" :"1993-04-22 11:10:00.000"},
  : "1", "targetId" : "GOES-07", "startDateTime" : "1993-05-21 08:08:00.000", "endDateTime" :"1993-05-21 08:08:00.000"},
  : "1", "targetId" : "GOES-07", "startDateTime" : "1993-06-07 05:17:00.000", "endDateTime" :"1993-06-07 05:17:00.000"},
  : "1", "targetId" : "GOES-07", "startDateTime" : "1993-06-09 09:11:00.000", "endDateTime" :"1993-06-09 09:11:00.000"},
  : "1", "targetId" : "GOES-07", "startDateTime" : "1993-07-14 08:51:00.000", "endDateTime" :"1993-07-14 08:51:00.000"},
  : "1", "targetId" : "GOES-07", "startDateTime" : "1993-08-18 17:08:00.000", "endDateTime" :"1993-08-18 17:08:00.000"},
```

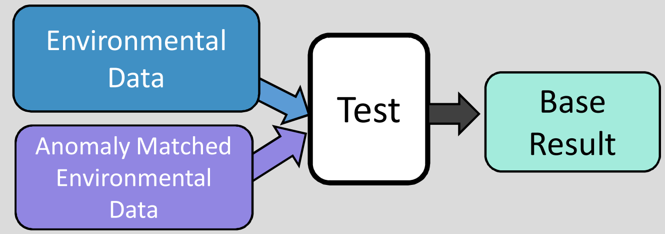


```
"dataTargets":[
  "GOES-07"
],
"geoTargetLocations" : [
  {
    "targetId":"GOES-07",
    "launchDateTime" : "1991-01-01 00:00:00.000",
    "locations":[
      {
        "startDateTime":"1991-01-01 00:00:00.000",
        "endDateTime":"1994-12-31 23:59:00.000",
        "westLongitude":210.0
      }
    ]
  }
```

SEAC in Action (2/4): The Process

1. Each Statistical Method Generates a Test Statistic ("Base Result")

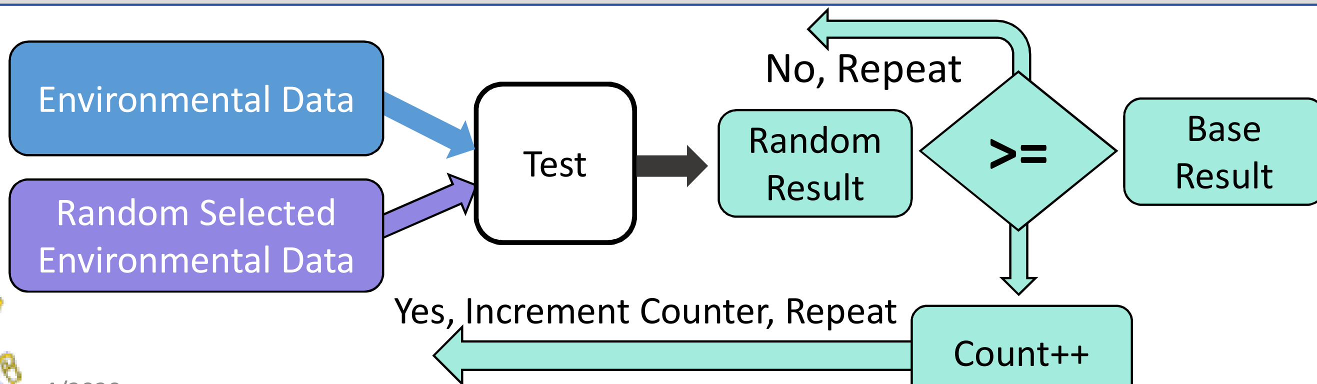
2. We calculate a Probability Value ("p-value") using Permutation Testing



p-value = Final Count / Bootstrap Iterations

- An assumption free probability ranging from 0 to 1
- How likely we are to observe a result at least as extreme as the one produced by the initial anomaly set

3. Control Family-Wise Error Rate (FWER) and False Discovery Rate (FDR)
   - Compare p-value to corresponding critical values to determine significance



SEAC in Action (3/4): The Outputs

Anomaly Analysis Report

```
---- [Anomaly Analysis Report] ----
Total Number : 57
Time Span : [ 1991-03-28 13:49:00.000Z, 1994-03-25 09:17:00.000Z ]
Discrete Timestamps : 57

---- [Anomalies By Target] ----
Total number of targets : 1
GOES-07 : 57

---- [Solar Cycle] ----
Warning! The anomalies for this analysis only span roughly 2.994003 years
meaning that it is possible that only a single period of solar
minimum is represented.

---- [Anomalies By Year] ----
1991 : 2
1992 : 12
1993 : 31
1994 : 12

---- [Anomalies By Month] ----
January : 5
February : 7
March : 9
April : 2
May : 3
June : 2
July : 2
August : 4
September : 7
October : 7
November : 4
December : 5

---- [Anomalies By Local Time] ----
Local time anomaly frequency binned by hour
0 : 1
1 : 0
2 : 0
3 : 1
4 : 0
5 : 0
6 : 0
7 : 0
8 : 1
9 : 0
10 : 0
11 : 0
12 : 0
13 : 1
14 : 2
15 : 1
16 : 1
17 : 1
18 : 11
19 : 7
20 : 10
21 : 9
22 : 6
23 : 5

---- [Equinox] ----
16 of 57 anomalies occur during equinox months (28.0702%)

---- [Duration Between Anomalies]
Maximum Duration in Days Between Anomalies : 409.149
Minimum Duration in Days Between Anomalies : 1.27222
Average Duration in Days Between Anomalies : 19.5145
```

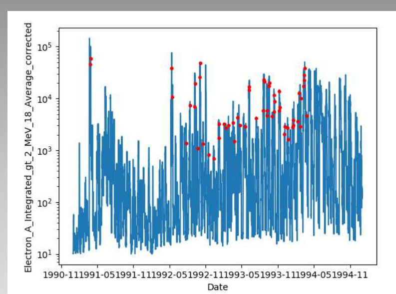
Driver Analysis Results

```
Driver, KOLMOGOROV_SMIRNOV_TEST
Electron_A_Integrated_gt_2_MeV_18_Average_corrected, 0.696981
Electron_A_Integrated_gt_2_MeV_18_Fluence_corrected, 0.700501
Electron_A_Integrated_gt_2_MeV_24_Average_corrected, 0.693081
Electron_A_Integrated_gt_2_MeV_24_Fluence_corrected, 0.689941
Electron_A_Integrated_gt_2_MeV_36_Average_corrected, 0.662485
Electron_A_Integrated_gt_2_MeV_36_Fluence_corrected, 0.663596
Electron_A_Integrated_gt_2_MeV_48_Average_corrected, 0.657655
Electron_A_Integrated_gt_2_MeV_48_Fluence_corrected, 0.656611
Proton_A_Integrated_gt_5_MeV_36_Average_corrected, 0.143283
Proton_A_Integrated_gt_5_MeV_48_Average_corrected, 0.141554
Proton_A_Integrated_gt_5_MeV_72_Average_corrected, 0.154249
Proton_A_Integrated_gt_5_MeV_96_Average_corrected, 0.126474
SingleEventEffects_Proton_A_Integrated_gt_10_MeV_corrected, 0.0192939
SingleEventEffects_Proton_A_Integrated_gt_30_MeV_corrected, 0.0593347
```

NOAA SPE Event List

```
# Proton_Integrated_gt_10_MeVA corrected Particle Event List
# Start Date, End Date, Max Date, Max Value
1991-01-31 11:30:00.000,1991-02-01 18:40:00.000,1991-01-31 16:20:00.000,242
1991-02-25 12:10:00.000,1991-02-25 13:40:00.000,1991-02-25 13:05:00.000,12.8
1991-03-23 08:20:00.000,1991-03-30 21:40:00.000,1991-03-24 03:50:00.000,43500
1991-04-03 08:15:00.000,1991-04-06 06:45:00.000,1991-04-04 10:00:00.000,52.4
1991-05-13 03:00:00.000,1991-05-14 06:30:00.000,1991-05-13 09:05:00.000,358
1991-05-31 12:25:00.000,1991-06-03 12:35:00.000,1991-06-01 05:00:00.000,22.9
1991-06-04 08:20:00.000,1991-06-18 20:45:00.000,1991-06-11 14:20:00.000,3010
1991-06-30 05:55:00.000,1991-07-05 11:15:00.000,1991-07-02 10:10:00.000,106
1991-07-07 04:55:00.000,1991-07-11 11:55:00.000,1991-07-08 16:50:00.000,2310
1991-07-12 00:00:00.000,1991-07-12 09:35:00.000,1991-07-12 03:05:00.000,17.1
1991-08-26 17:40:00.000,1991-08-28 22:25:00.000,1991-08-27 18:30:00.000,236
1991-10-01 17:40:00.000,1991-10-01 23:15:00.000,1991-10-01 18:10:00.000,12.4
1991-10-28 13:00:00.000,1991-10-28 15:50:00.000,1991-10-28 14:40:00.000,39.8
1991-10-30 07:45:00.000,1991-10-31 18:30:00.000,1991-10-30 08:10:00.000,94
1992-02-07 06:45:00.000,1992-02-08 21:25:00.000,1992-02-07 11:15:00.000,77.8
1992-03-16 08:40:00.000,1992-03-16 10:05:00.000,1992-03-16 09:00:00.000,10.4
1992-05-09 10:05:00.000,1992-05-11 08:45:00.000,1992-05-09 21:00:00.000,4650
1992-06-25 20:45:00.000,1992-06-29 16:05:00.000,1992-06-26 06:15:00.000,389
1992-08-06 11:45:00.000,1992-08-06 20:35:00.000,1992-08-06 12:10:00.000,14.2
1992-10-30 19:20:00.000,1992-11-06 01:20:00.000,1992-10-31 07:10:00.000,2680
1993-03-04 15:05:00.000,1993-03-04 20:00:00.000,1993-03-04 16:15:00.000,17.1
1993-03-12 20:10:00.000,1993-03-13 12:40:00.000,1993-03-13 01:55:00.000,44
1994-02-20 03:05:00.000,1994-02-22 00:50:00.000,1994-02-21 09:00:00.000,10200
1994-10-20 00:30:00.000,1994-10-20 12:20:00.000,1994-10-20 03:40:00.000,34.9
```

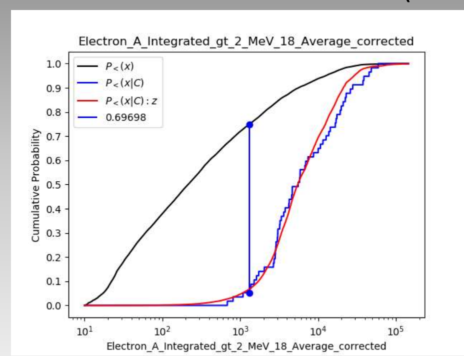
Statistical Quantities for Every Candidate Driver Displayed for further evaluation by user



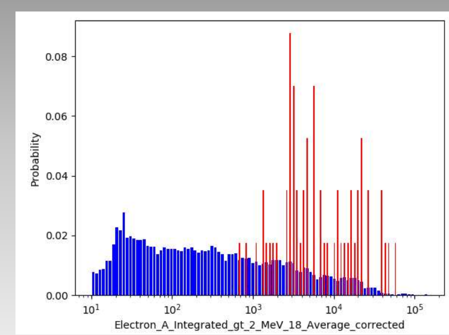
Reference Distribution (Blue)

Anomaly Distribution (Red)

Cumulative Distribution Functions (CDFs)



Probability Mass Function (PMF)



U.S. AIR FORCE

SEAC in Action (4/4): The Decision Logic

```
Driver Description, P-value, Holm-Bonferroni FWER Control, Benjamini-Hochberg FDR Control, Accept/Reject Candidate
Electron_A_Integrated_gt_2_MeV_18_Fluence_corrected,0,0.00357143,0.00357143,ACCEPT
Electron_A_Integrated_gt_2_MeV_18_Average_corrected,0,0.00384615,0.00714286,ACCEPT
Electron_A_Integrated_gt_2_MeV_24_Average_corrected,0,0.00416667,0.0107143,ACCEPT
Electron_A_Integrated_gt_2_MeV_24_Fluence_corrected,0,0.00454545,0.0142857,ACCEPT
Electron_A_Integrated_gt_2_MeV_36_Fluence_corrected,0,0.005,0.0178571,ACCEPT
Electron_A_Integrated_gt_2_MeV_36_Average_corrected,0,0.00555556,0.0214286,ACCEPT
Electron_A_Integrated_gt_2_MeV_48_Average_corrected,0,0.00625,0.025,ACCEPT
Electron_A_Integrated_gt_2_MeV_48_Fluence_corrected,0,0.00714286,0.0285714,ACCEPT
Proton_A_Integrated_gt_5_MeV_72_Average_corrected,0.058,0.00833333,0.0321429,REJECT
Proton_A_Integrated_gt_5_MeV_36_Average_corrected,0.084,0.01,0.0357143,REJECT
Proton_A_Integrated_gt_5_MeV_48_Average_corrected,0.11,0.0125,0.0392857,REJECT
Proton_A_Integrated_gt_5_MeV_96_Average_corrected,0.146,0.0166667,0.0428571,REJECT
SingleEventEffects_Proton_A_Integrated_gt_30_MeV_corrected,0.624,0.025,0.0464286,REJECT
SingleEventEffects_Proton_A_Integrated_gt_10_MeV_corrected,0.948,0.05,0.05,REJECT
```

Eliminated Candidates

```
# Candidate, Statistic, Number of Reasons for Elimination, Reasons for Elimination
Proton_A_Integrated_gt_5_MeV_72_Average_corrected, 0.154249,2, Only 2 out of 57 occurred during a proton event. p-value for analysis was not significant.
Proton_A_Integrated_gt_5_MeV_36_Average_corrected, 0.143283,2, Only 2 out of 57 occurred during a proton event. p-value for analysis was not significant.
Proton_A_Integrated_gt_5_MeV_48_Average_corrected, 0.141554,2, Only 2 out of 57 occurred during a proton event. p-value for analysis was not significant.
Proton_A_Integrated_gt_5_MeV_96_Average_corrected, 0.126474,2, Only 2 out of 57 occurred during a proton event. p-value for analysis was not significant.
SingleEventEffects_Proton_A_Integrated_gt_30_MeV_corrected, 0.0593347,2, Only 2 out of 57 occurred during a proton event. p-value for analysis was not significant.
SingleEventEffects_Proton_A_Integrated_gt_10_MeV_corrected, 0.0192939,2, Only 2 out of 57 occurred during a proton event. p-value for analysis was not significant.
```

Remaining Candidates

```
# Candidate, Statistic
Electron_A_Integrated_gt_2_MeV_18_Fluence_corrected, 0.700501
Electron_A_Integrated_gt_2_MeV_18_Average_corrected, 0.696981
Electron_A_Integrated_gt_2_MeV_24_Average_corrected, 0.693081
Electron_A_Integrated_gt_2_MeV_24_Fluence_corrected, 0.689941
Electron_A_Integrated_gt_2_MeV_36_Fluence_corrected, 0.663596
Electron_A_Integrated_gt_2_MeV_36_Average_corrected, 0.662485
Electron_A_Integrated_gt_2_MeV_48_Average_corrected, 0.657655
Electron_A_Integrated_gt_2_MeV_48_Fluence_corrected, 0.656611
```

Ongoing efforts and challenges…

- Current SEAC analyses (in progress):
  - Use SCATHA, CRRES data, and anomaly lists for continued development
  - Quantify the uncertainty in attributing data from 'nearby' satellite to target (when target data not available)
- Challenge in obtaining:
  - Anomaly lists from targets of interest
  - Complete environmental datasets that span mission duration for targets of interest
- Awaiting publicly available datasets:
  - GOES 16 MPS-Lo flux data and Plasma Driver development

Conclusions and road ahead for SEAC

- Availability/release timescale of SEAC application to stakeholders
  - Based on user requirements
  - Space environment data backend
- Integration into existing, operational tools
- New developments
  - Integrated Anomaly Toolkit
    - In concert with the SatCat tool (Janet Green of Space Hazard Applications)
    - Consultants include Aerospace (Paul O’Brien), AER (Rick Quinn)
  - Seeking partnerships, external investments

References

[1] O'Brien, T. P., SEAS-GEO: A spacecraft environmental anomalies expert system for geosynchronous orbit, Space Weather, vol. 7, S09003, doi:10.129/2009SW000473, 2009.

[2] Wrenn, G. L., Conclusive evidence for internal dielectric charging anomalies on geosynchronous communications spacecraft, J. Spacecraft and Rockets, vol. 32, No. 3, 1995.

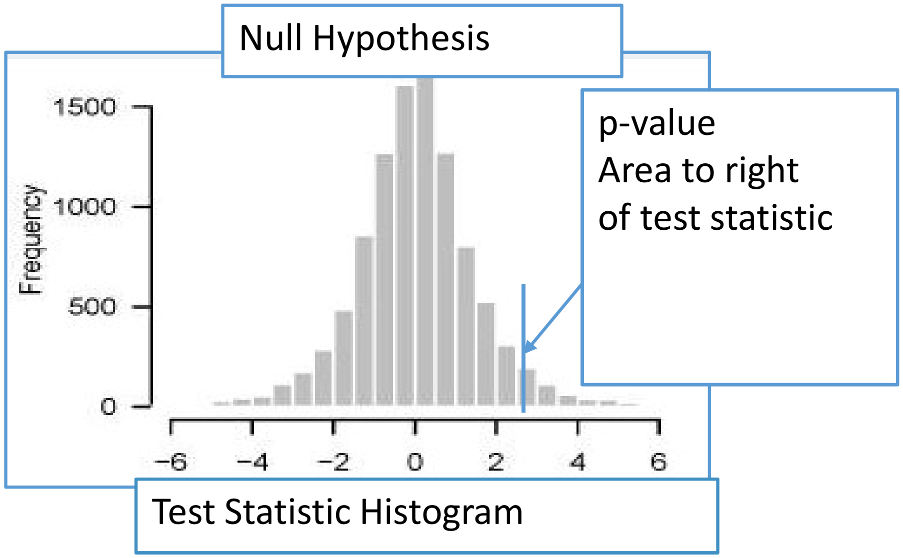
[3] Wrenn, G. L., D. J. Rodgers, and K. A. Ryden, A solar cycle of spacecraft anomalies due to internal charging, Annales Geophysicae, 20: 953-956, 2002.

[4] Y.-J. Su, J. M. Quinn, W. R. Johnston, J. P. McCollough and M. J. Starks, "Specification of > 2 MeV electron flux as a function of local time and geomagnetic activity at geosynchronous orbit," *Space Weather,* pp. 470-486, 2014.

Additional Slides

Hypothesis Testing and Probability Value (p-value)

- The probability of observing a value (or a more extreme one) when those values are drawn from the known, and fixed distribution (i.e., null is true).
  - Probability values are uniformly distributed between 0 and 1 under Null Hypothesis
  - P-value histogram (and/or QQ Plot) can provide information on how 'well-behaved" p-values are when performing multiple hypothesis tests (and if there is sufficient "power" in the given test)
- Before testing, determine a significance threshold "alpha" (for example, 5%) below which p-values are significant ("Reject" the Null)



Ref:
https://www.ncbi.nlm.nih.gov/pmc/articles/PMC6164648

U.S. AIR FORCE

stellar science

Interpreting p-value: no single index should substitute for scientific reasoning

- “P < 0.05” Might Not Mean What You Think: American Statistical Association Clarifies P Values https://www.ncbi.nlm.nih.gov/pmc/articles/PMC5017929/
- Advice from ASA Panel:
  - Fully report every aspect of the study (transparency).
    - Specifics of data collection and curation are critical for inference (were data deleted because they seemed to be ‘outliers’?)
  - Consult a statistician when writing a grant application rather than after the study is finished
  - Limit the number of hypotheses to be tested to a realistic number that doesn’t increase the false discovery rate
  - Be conservative in interpreting the data
  - Don’t consider P = 0.05 as a magic number
  - And whenever possible, provide confidence intervals

P-value Using Permutation Testing

- Produce an assumption free P-value that can tell us whether the results of a given test are significant
- Previously, we relied on critical values that are either based off a table or calculated from a formula that inherently have assumptions such as the type of test, etc.
- Permutation Testing is now used with any of the statistical tests in SEAC
- Procedure:
  - Perform test as usual where X is the environmental distribution and Y is the anomaly time matched distribution producing result R1
  - Randomly shuffle the anomaly times with times from the environmental distribution to produce a random sample Y_shuffled and repeat the test using this random sampling, producing R_shuffled.
  - Repeat at least 500 times so that we have a set of results R_shuffled_1 – R_shuffled_500.
  - We can define our P-value as (the number of R_shuffled_ results that are >= R1) / 500
  - This gives us an assumption free probability ranging from 0 to 1 that tells us how likely we are to observe a result at least as extreme as the one produced by the initial anomaly set.

Controlling FWER and FDR

- Family-Wise Error Rate control:

  Holm-Bonferroni
  - Given n tests, put p-values in ascending order
  - Specify a target significant level (for example, 5%)

    $$\alpha = 0.05$$
  - Determine the critical value for each p-value, p_i

    $$\frac{\alpha}{n - i + 1}$$

Compare and exclude those p-values that are greater than their corresponding critical values as they are *not* significant

- False Discovery Rate control:

  Benjamini-Hochberg
  - Given n tests, put p-values in ascending order
  - Specify a False Discovery Rate (for example, 25%)

    FDR = 0.25
  - Determine the critical value for each p-value, p_i

    $$\frac{i \times FDR}{n}$$

Compare and include those p-values that are less than their corresponding critical values

Hazard Quotient Calculation

Hazard Quotient: A continuous function **z(x)**, that when multiplied by the total probability of an anomaly for all **x**, gives the probability of that anomaly at any **x**.

For Example: If we knew the total probability **P** of an anomaly due to 2MeV electrons, the hazard quotient **z(x)** would allow us to calculate the probability of an anomaly (due to 2MeV electrons) at any fluence **x** as **P(x) = z(x) * P**

- Code provided by AFRL refactored and integrated into SEAC
  - Computes a non-linear least-squares fit to an assumed power law form, **z(x) = x** ^{gamma}
  - Tested against analytical case and the provided AFRL code
- SEAC computes the hazard quotient for significant drivers

Known Driver Distributions

$$p(x|C) = [(1 - c_x) + c_x z(x)]p(x)$$

Known Anomaly Distributions

$$P_<(x|C) = (1 - c_x)P_<(x) + c_x \frac{\int_{-\infty}^{x} \hat{z}(x')p(x')dx'}{\langle \hat{z}(x) \rangle}$$

Compute cx and z(x) from Least Squares

*O'Brien, T. P. "SEAES-GEO: A spacecraft environmental anomalies expert system for geosynchronous orbit." *Space Weather* 7.9 (2009)

U.S. AIR FORCE

stellar science